STUDY OF HEAT AND MASS TRANSFER IN PIPE FLOWS WITH NON-NEWTONIAN FLUIDS

J. T. TENG,† R. GREIF, I. CORNET and R. N. SMITH‡

University of California, Berkeley, Department of Mechanical Engineering,
Berkeley, CA 94720, U.S.A.

Abstract—Experimental results are reported for the mass transfer in pipe flow for an aqueous 4% sodium chloride solution under Newtonian and non-Newtonian conditions. The non-Newtonian runs were carried out with concentrations of 50 and 1000 wppm of Polyox WSR 301. For the non-Newtonian flows there is a significant reduction in the mass transfer.

NOMENCLATURE

A, constant;
A_e, area of electrode;
c, concentration;
c_p, specific heat;
d, diameter;
D, diffusivity;
f, friction coefficient;
h, transfer coefficient;
k, thermal conductivity;
K, viscometric property;
L, length;
$\dot{m}$, mass flux;
n, viscometric property;
q, heat flux;
r, radial coordinate;
R, radius;
u, velocity;
u_τ, friction velocity;
T, temperature;
y, radial coordinate measured from wall;
y^+, dimensionless coordinate, yu_τ/ν;
Nu, Nusselt number;
Pr^*, generalized Prandtl number;
Re^*, generalized Reynolds number;
Sc, Schmidt number;
Sc^*, generalized Schmidt number;
Sh, Sherwood number.

Greek symbols

α, thermal diffusivity;
ε, eddy diffusivity;
κ, constant;
μ, viscosity;
ρ, density;
τ, shear stress.

Subscripts

b, bulk;
t, turbulent;
w, wall;
CL, center line;
H, heat;
M, momentum.

INTRODUCTION

IN THE present work experimental results are reported for mass transfer in pipe flow for aqueous 4% sodium chloride solution containing concentrations of 0, 50 and 1000 wppm of a long chain polymer; namely, Polyox WSR 301 (Union Carbide), a completely water soluble polymer of ethylene oxide. A fluid is rendered non-Newtonian by this or similar additives [1–17] and the present results show that this is accompanied by a marked reduction in the convective mass transport. The experimental data are compared with theoretical and semiempirical relations for the mass transfer.

EXPERIMENTAL SYSTEM

The experimental system consisted of the recirculating loop, test section, pump, control valves and storage tank, which are shown schematically in Fig. 1. A long entry section was used (125 diameters) to obtain a fully developed velocity profile at the test section. All the valves, fittings and pipes that were used were made from polyvinyl chloride (PVC) to resist corrosion from salt solutions. The fluids used were 4% sodium chloride aqueous solutions, which were rendered non-Newtonian by addition of various concentrations of Polyox WSR 301.

The fluid was circulated by a one-stage screw type Moyno pump, model CSQ 1L10H, with a capacity of 168 gal/min. The pump housing was made of cast iron and was coated with a layer of Bitumastic No. 300-M (Koppers Co.) to protect the surface from the corrosive salt solution. In addition, the pump was

† General Electric Company, San Jose, California, U.S.A.
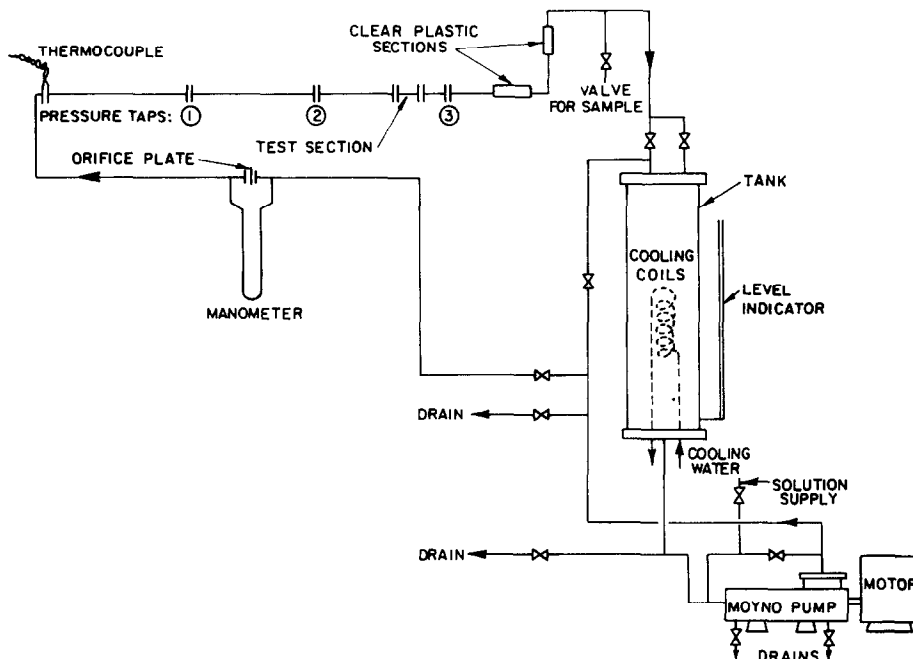‡ Rensselaer Polytechnic Institute, Troy, New York, U.S.A.

FIG. 1. Schematic diagram of flow system.

cathodically protected by the insertion of three zinc plugs in the housing. The flow rate was determined from a pressure drop measurement across a sharp-edged orifice plate and the temperature of the fluid was determined from a thermocouple that was immersed in the stream. The concentration of the dissolved oxygen in the fluid was measured by the Winkler technique [28] and by a Beckman pH-meter with an oxygen adaptor. The viscosity was obtained from measurements that were made with a Fann model V-G viscometer and the diffusivity from measurements that were previously reported [12].

An electrochemical cell was established with a monel 400 alloy section of the pipe serving as the cathode. The anode was a platinized titanium pipe, 7.6 cm in length, that was inserted downstream of the cathode. Individual cathode sections of lengths 0.063, 0.301, 0.974, 2.516 and 3.810 cm were made and a "spool" was assembled (cf. van Shaw et al. [18]) containing the five monel discs which were insulated from one another by discs of acrylic plastic, 1.3 cm in thickness (cf. Fig. 2). Sealing was established by 0-rings that were set into the plastic. All of these pieces were bored individually to an inner diameter of 2.42 cm and were assembled with non-conducting aligning pins and steel bolts (insulated from the monel) to keep the unit rigid and insure that no discontinuities arose on the inner surface. The unit was sanded to a smooth finish. The cathode spool was then inserted into place and any of the individual monel sections could then be used as the electrode. The spool was bolted between external flanges and sealed with 0-rings.

Each plastic insulation section contained a hole drilled for placement of a silver–silver chloride reference electrode. The hole was tapered and had a

0.01 in capillary opening to provide a connection between the test fluid and the reference electrode. The reference electrode was placed in the hole immediately downstream of the operating cathode and the other holes in the spool were plugged.

Measurements were made of the limiting current associated with the reduction of dissolved oxygen at
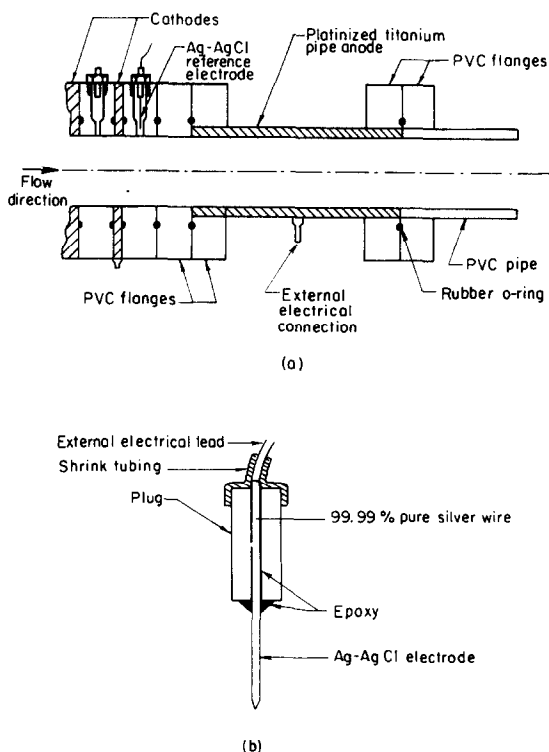

FIG. 2. (a) Schematic diagram of test section. (b) Details for reference electrode.
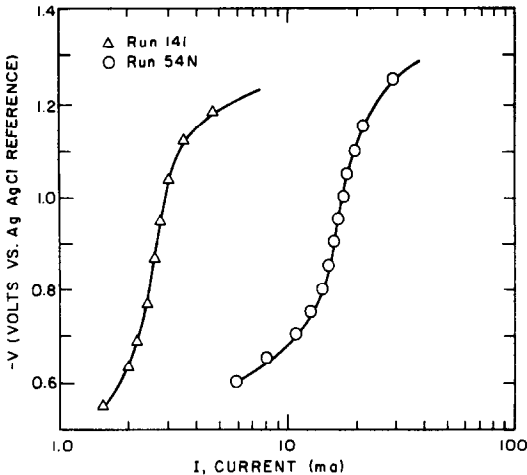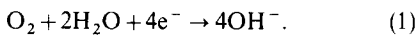
FIG. 3. Typical limiting current curves. Run 141, Newtonian fluid, $Re = 39\,800$, $Sc = 415$, $L^+ = 261$, $L/d = 0.125$. Run 54N, non-Newtonian fluid, 1000 wppm of Polyox WSR 301, $Re^* = 14\,400$, $Sc^* = 1170$, $L^+ = 265$, $L/d = 1.576$.

the cathode surface according to the net reaction:

$$O_2 + 2H_2O + 4e^- \rightarrow 4OH^-. \tag{1}$$

The limiting current was obtained from a galvanostatic plot of the potential of the cathode surface relative to a reference voltage vs the current. Typical polarization curves are shown in Fig. 3. The point of the inflection on the polarization curve occurs at the mass-transfer limiting current, this point corresponding to zero oxygen concentration at the metal surface.

COMPARISON OF RESULTS AND DISCUSSION

When the potential difference is sufficiently large the concentration of oxygen at the cathode surface is

essentially zero and the current is limited by the diffusional mass transfer. From Faraday's law and the definition of the mass-transfer coefficient we obtain:

$$h \equiv \frac{\dot{m}}{c_b - c_w} = \frac{I_L(1-t)}{n_e F A_e c_b}, \tag{2}$$

where I_L is the limiting current, n_e is the number of electrons (4) exchanged in the reaction, A_e is the area of the electrode and t is the transference number. Because of the excess of sodium chloride, t is much less than unity and ionic migration effects are negligible. To check the experimental apparatus a series of measurements were first carried out for the Newtonian condition; that is, in the absence of Polyox and the results for the 4% sodium chloride aqueous solution are presented in Fig. 4. A comparison of these data for Newtonian fluids and small values of L^+, that is, the entry region mass-transfer problem, can be analyzed according to the work of Lighthill and Leveque [19, 20] and the result is given by [18]

$$\frac{h}{u_\tau} = \frac{\dot{m}}{(c_b - c_w)u_\tau} = 0.81 Sc^{-2/3}(L^+)^{-1/3}. \tag{3}$$

Note that this relation is valid for laminar or turbulent flows with the correspondingly different values for $u_\tau = u_b(f/2)^{1/2}$ incorporated into $L^+ = Lu_\tau/\nu$. It is seen in Fig. 4 that there is good agreement between the experimental data and the predictions for small values of L^+.

For large values of L^+ the concentration profile is fully developed and the mass-transfer coefficient is independent of the length. For this case the result for the mass-transfer coefficient is given by (van Driest [21], Deissler [22])

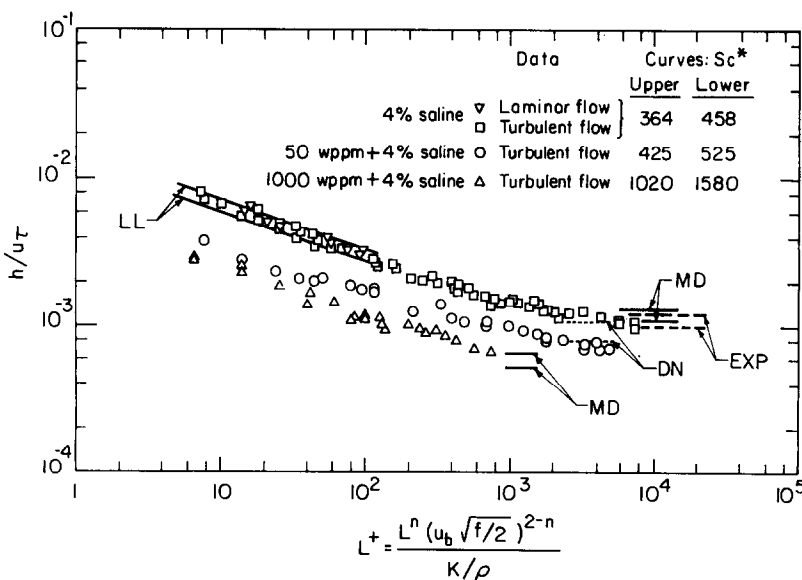$$\frac{h}{u_\tau} = 0.11 Sc^{-3/4} Sc_t^{-1/4}, \tag{4}$$

FIG. 4. Mass-transfer results (LL, Lighthill-Leveque; MD, Modified Diffusivity; EXP, Expansion; DN, Dilute Non-Newtonian).

or by (Tien, Wasan and Wilke [23-25])

$$\frac{h}{u_\tau} = 0.062 Sc^{-2/3} Sc_t^{-1/3}. \tag{5}$$

These results are in good agreement with the experimental data as presented in Fig. 4. A value of Sc_t equal to unity was taken.

Experiments were also carried out for non-Newtonian flows and these data are presented in Fig. 4 for concentrations of 50 and 1000 wppm of Polyox WSR 301. The specification of L^+ in Fig. 4 is in the form given by Wilkinson [29] and Skelland [30]. For the 50 wppm concentration n is approximately unity and $K(kg/m\,s^{2-n})$ varies from 0.00098 to 0.00109; for 1000 wppm the values of n vary from 0.72 to 0.79 while k varies from 0.0083 to 0.0123. It is noted that no variations were observed during a run except for those resulting from a slight temperature rise that was sometimes noted. This is in contrast to the case with a concentration of 3000 wppm of Polyox. For this case the solution degraded continuously during the run [26] and these results have not been presented.

For dilute concentrations of non-Newtonian additives, McConaghy and Hanratty [17] present the following two semi-empirical relations for the fully developed mass-transfer coefficient:

$$\frac{h}{u_\tau} = 0.900 Sc^{-3/4}\left(\frac{0.40}{A_C^+}\right)^{1/2} Sc_t^{-1/4} \tag{6}$$

and

$$\frac{h}{u_\tau} = 0.827 Sc^{-2/3}\left(\frac{0.40}{[A_L^+]^2}\right)^{1/3} Sc_t^{-2/3}. \tag{7}$$

The values of A_C^+ and A_L^+ are given by McConaghy [16]. For the experiments with the 50 wppm concentration of Polyox WSR 301, the range of values for the Reynolds number was from 3800 to 86 600, the Schmidt number from 425 to 525, L^+ from 7.4 to 4800 and A_C^+ and A_L^+ from 30 to 40.† The results obtained from equations (6) and (7) were quite close and are slightly larger than the experimental data and are designated as DN (dilute non-Newtonian) in Fig. 4. A value of Sc_t equal to unity was chosen.

For moderately large concentrations of a non-Newtonian additive an alternative procedure is needed to predict the mass transfer. One approach is to modify the eddy diffusivity relation of van Driest [21] and Deissler [22] (also refer to McKillop [27]) in conjunction with an equivalent viscosity, $\mu_w \equiv K(|du/dr|)^{n-1}$. Following this procedure the following relation has been obtained by Teng [26] for the velocity profile:

and the eddy diffusivity is given by:

$$\varepsilon_M = (\kappa y)^2[1 - \exp(-yu_\tau\rho/\mu_w A)]^2(du/dy). \tag{9}$$

Based on Clapp's velocity profile data [4, 5] for values of n between 0.698 and 0.786 and values of $K(kg/m\,s^{2-n})$ between 0.039 and 0.1178 the constants κ and A were taken to be 0.37 and 30.0, respectively.

An alternative procedure for the moderate concentration condition is to use the expansion method of Tien, Wasan and Wilke [23, 25] in conjunction with the measurements of Clapp [4, 5]. Omitting the details, the results for the velocity profile near the wall are given by Teng [26]:

$$u^+ = (y^+)^{1/n} + u_4^+(y^+)^{4/n} + u_5^+(y^+)^{5/n} \tag{10}$$

and the eddy diffusivity is given by:

$$\varepsilon_M = -n\left(\frac{K}{\rho}\right)^{1/n} u_\tau^{2(n-1)/n}$$
$$\times \frac{4u_4^+(y^+)^3 + 5u_5^+(y^+)^4}{1 + 4u_4^+(y^+)^3 + 5u_5^+(y^+)^4} \tag{11}$$

with $u_4^+ = -2.95 \times 10^{-4}$ and $u_5^+ = 1.4 \times 10^{-5}$. The values chosen for u_4^+ and u_5^+ are based on the velocity profiles given by Clapp [4, 5].

For fully developed turbulent pipe flow the temperature profile, with $Pr_t = 1$ and the constant flux assumption is given by:

$$T^+ \equiv \frac{T_w - T}{q_w/\rho c_p u_\tau} = \frac{1}{n}\left(\frac{K}{\rho}\right)^{1/n}\frac{u_\tau^{2-(2/n)}}{\alpha}$$
$$\times \int_0^{y^+}\frac{(y^+)^{(1/n)-1}\,dy^+}{\left(1 - \frac{y^+}{R^+}\right)\left(1 + \frac{\varepsilon_H}{\alpha}\right)}. \tag{12}$$

The Nusselt number, Nu, based on

$$T_b^+ = \frac{T_w - T_b}{q_w/\rho c_p u_\tau}$$

is given by:

$$Nu = \frac{hd}{k} = \frac{1}{T_b^+} Re^* Pr^* (f/2)^{1/2} \tag{13}$$

where

$$Re^* = \frac{8\rho u_b^2}{K\left(\frac{8u_b}{d}\right)^n\left(\frac{3n+1}{4n}\right)^n}$$

$$\text{and } Pr^* = \frac{c_p K}{k}\left(\frac{3n+1}{4n}\right)^n\left(\frac{8u_b}{d}\right)^{n-1} \tag{14}$$

Equation (13) was obtained by integrating equation (12) in conjunction with equations (8) and (9).

$$u^+ = \frac{u}{u_\tau} = \frac{1}{u_\tau}\int_0^y \frac{2\,dy}{\frac{\mu_w}{\tau_w} + \left\{\left(\frac{\mu_w}{\tau_w}\right)^2 + \frac{4\rho}{\tau_w}(\kappa y)^2\left[1 - \exp\left(\frac{-yu_\tau\rho}{\mu_w A}\right)\right]^2\right\}^{1/2}} \tag{8}$$

† The referee has noted that there are large uncertainties in equations (6) and (7).

Comparison of the results obtained from the modified diffusivity method with the measured temperature profiles of Clapp with Carbopol 934 give a maximum disagreement of 8% in the center-line temperature, T_{CL}^+. For the expansion method the predicted results yield a maximum disagreement of 17%. Comparable differences are obtained for comparisons that were made for the Nusselt number.

Calculations were also carried out to determine the mass transfer for the case of the 1000 wppm concentration of Polyox WSR 301. For mass transfer we have:

$$Sh = \frac{hd}{D} = \frac{1}{c_b^+} Re^* Sc^* (f/2)^{1/2} \qquad (15)$$

with

$$Sc^* = \frac{K}{\rho D} \left(\frac{3n+1}{4n}\right)^n \left(\frac{8u_b}{d}\right)^{n-1} \qquad (16)$$

where the value of the diffusivity, D, is reported in [12]. The data cover the range for Reynolds numbers from 6000 to 55 000, for Schmidt numbers from 1020 to 1580 and for L^+ from 6.7 to 750. A comparison in the fully developed region indicates good agreement between the modified diffusivity method and the data (cf. Fig. 4). However, the highest value of L^+ attainable with the present experimental system for the 1000 wppm concentration of Polyox WSR 301 is only 750, and additional data are required to appraise the results. For this condition, the expansion method was not utilized.

In summary, experimental results have been reported for mass transfer in pipe flow. For an aqueous 4% sodium chloride solution the results are in good agreement with theoretical predictions. With the addition of concentrations of 50 and 1000 wppm of Polyox WSR 301 there is a marked reduction in the mass transfer. For the 50 wppm concentration the data in the fully developed region are in good agreement with the relations presented by McConaghy and Hanratty [17]. For the 1000 wppm concentration, the work of previous investigators has been modified and there is an indication of good agreement.

REFERENCES

1. A. Metzner, R. Vaughn and G. Houghton, Heat transfer to non-Newtonian fluids, A.I.Ch.E. Jl 3, 92–100 (1957).
2. A. Metzner and D. Gluck, Heat transfer to non-Newtonian fluids under laminar flow conditions, Chem. Engng Sci. 12, 185–190 (1960).
3. S. B. Clough, H. E. Read, A. B. Metzner and V. C. Behn, Diffusion in slurries and in non-Newtonian fluids, A.I.Ch.E. Jl 8, 346–350 (1962).
4. R. M. Clapp, Ph.D. Dissertation, University of Cambridge, Cambridge, England (1960).
5. R. M. Clapp, Turbulent heat transfer in pseudoplastic non-Newtonian fluids, in International Developments in Heat Transfer, p. 652, D211. A.S.M.E., New York (1963).
6. G. Astarita, G. Marrucci and G. Palumbo, Non-Newtonian gravity flow along inclined plane surfaces, I/EC Fundamentals 3, 333–339 (1964).
7. G. Astarita, Diffusivity in non-Newtonian liquids, I/EC Fundamentals 4, 236–237 (1965).
8. G. Astarita, Mass transfer from a flat solid surface to a falling non-Newtonian liquid film, I/EC Fundamentals 5, 14–18 (1966).
9. M. K. Gupta, A. B. Metzner and J. P. Hartnett, Turbulent heat transfer characteristics of viscoelastic fluids, Int. J. Heat Mass Transfer 10, 1211–1224 (1967).
10. G. S. Hansford and M. Litt, Mass transport from a rotating disc into power-law liquids, Chem. Engng Sci. 23, 849–864 (1968).
11. H. J. Bader, A. A. McKillop and J. C. Harper, An experimental and analytical study of entrance flow of non-Newtonian fluids, in Fourth International Heat Transfer Conference, Paris, Vol. 4, pp. Rh 1–10 (1970).
12. R. Greif, I. Cornet and R. Kappesser, Diffusivity of oxygen in a non-Newtonian saline solution, Int. J. Heat Mass Transfer 15, 593–597 (1972).
13. R. Greif, R. Kappesser and I. Cornet, Transport in non-Newtonian flow, J. Electrochem. Soc. 119, 717–718 (1972).
14. J. A. Paterson, R. Greif and I. Cornet, Experimental and theoretical results for mass transfer to a rotating cone in a non-Newtonian solution, Int. J. Heat Mass Transfer 16, 1017–1024 (1973).
15. P. M. Debrule and R. H. Sabersky, Heat transfer and friction coefficients in smooth and rough tubes with dilute polymer solutions, Int. J. Heat Mass Transfer 17(5), 529–540 (1974).
16. G. A. McConaghy, The effect of drag reducing polymers on turbulent mass transfer, Ph.D. Dissertation, University of Illinois, Urbana (1974).
17. G. A. McConaghy and T. J. Hanratty, Influence of drag reducing polymers on turbulent mass transfer to a pipe wall, A.I.Ch.E. Jl 23(4), 493–500 (1977).
18. P. van Shaw, L. P. Reiss and T. J. Hanratty, Rates of turbulent transfer to a pipe wall in the mass transfer entry region, A.I.Ch.E. Jl 9(3), 362 (1963).
19. M. J. Lighthill, Contributions to the theory of heat transfer through a laminar boundary layer, Proc. R. Soc. 202A, 359–377 (1950).
20. J. Leveque, Comptes Rendus 185, 1190 (1927).
21. E. R. van Driest, On turbulent flow near a wall, J. Aero. Sci. 23, 1007 (1956).
22. R. G. Deissler, Analysis of turbulent heat transfer, mass transfer and friction in smooth tubes at high Prandtl and Schmidt numbers, N.A.C.A. Report 1210 (1955).
23. C. L. Tien and D. T. Wasan, Law of the wall in turbulent channel flow, Physics of Fluids 6(1), 144 (1963).
24. D. T. Wasan, C. L. Tien and C. R. Wilke, Theoretical correlation of velocity and eddy viscosity for flow close to a pipe wall, A.I.Ch.E. Jl 9(4), 567 (1963).
25. C. L. Tien, A note on the distributions of temperature and eddy diffusivity for heat in turbulent flow near a wall, J. Appl. Math. Phys. 15(1), 63 (1964).
26. J. T. Teng, Experimental and theoretical studies in convective transport, Ph.D. Dissertation, University of California, Berkeley (1978).
27. A. A. McKillop, Prediction method for turbulent momentum and heat transfer in viscous non-Newtonian liquids, in Progress in Heat Mass Transfer, Vol. 15, p. 263 (1972).
28. W. W. Scott, Standard Methods of Chemical Analysis, Vol. 2, 5th Edn, pp. 2079–2081. Van Nostrand, New York (1939).
29. W. L. Wilkinson, Non-Newtonian Fluids—Fluid Mechanics, Mixing and Heat Transfer, Pregamon Press, New York (1960).
30. A. H. N. Skelland, Non-Newtonian Flow and Heat Transfer. John Wiley, New York (1967).

ETUDE DU TRANSFERT DE CHALEUR ET DE MASSE POUR DES ECOULEMENTS DANS DES TUBES DE FLUIDES NON-NEWTONIENS

Résumé—Des résultats expérimentaux sont donnés pour le transfert massique relatif à un écoulement en conduite d'une solution aqueuse de 4%₀ de chlorure de sodium dans des conditions Newtoniennes et non-Newtoniennes. Les essais non-Newtoniens sont réalisés avec des concentrations de 50 et 1000 ppm de Polyox WSR 301. Pour les écoulements non-Newtoniens on constate une réduction sensible du transfert massique.

EINE STUDIE ZUM WÄRME- UND STOFFÜBERGANG BEI ROHRSTRÖMUNGEN MIT NICHTNEWTONSCHEN FLÜSSIGKEITEN

Zusammenfassung—Es wird von Versuchsergebnissen über den Stoffübergang in einer Rohrströmung für eine wäßrige 4%—Natriumchloridlösung unter newtonschen und nichtnewtonschen Bedingungen berichtet. Die nichtnewtonschen Versuche wurden mit Polyox WSR 301 in Konzentrationen von 50 und 1000 ppm durchgeführt. Für die nichtnewtonschen Flüssigkeiten ergibt sich ein spürbarer Rückgang im Stoffübergang.

ИССЛЕДОВАНИЕ ТЕПЛО- И МАССОПЕРЕНОСА ПРИ ТЕЧЕНИИ НЕНЬЮТОНОВСКИХ ЖИДКОСТЕЙ В ТРУБЕ

Аннотация — Представлены результаты опытов по исследованию массообмена при ньютонов-ском и неньютоновском течении в трубе 4%-го водного раствора хлористого натрия. Не-ньютоновость течения создавалась введением в раствор добавок полиокса WSR 301 с кон-центрациями в 50 и 1000 миллионных долей (по весу). При этом наблюдалось значительное ослабление массопереноса.